Radiation Heat-Transfer Model for Fibers Oriented Parallel to Diffuse Boundaries

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A rigorous derivation of the two-flux parameters is presented for the fibrous medium with fibers oriented parallel to diffuse planar boundaries. The analysis accounts for the fiber orientation and the effect of diffuse radiation on the scattering characteristics of this type of fibrous medium. Parametric studies were performed to show the effect of optical properties and size parameters of the fibers on the backscatter factor and the net radiative heat flux through the medium.

Nomenclature

B	= backward parameter, defined by Eq. (15b)
$\bar{B}$	= backscatter factor, defined by Eq. (19c)
C	= cross section
d^2F	= distribution function
e_1,e_2	= emissive powers of the boundaries
F	= forward parameter, defined by Eq. (15a)
f_v	= volume fraction of fibers
i	= unit intensity function
I	= radiation intensity
j	= $\sqrt{-1}$
K	= extinction coefficient
L	= depth of medium
l	= average length of fiber
M	= total number of weave directions
$\mathbf{m}$	= complex index of refraction
N	= number density of fibers
$N(r)$	= fiber size distribution
p	= scattering phase function
q	= radiative heat flux
Q	= efficiency
r	= radius of cylinder
x_j	= fraction of fibers oriented in the jth direction
y	= depth along medium
β_a	= absorption ratio, defined by Eq. (19b)
δ	= Kronecker delta function
ε	= surface emisivity
θ	= angle of observation
θ_c	= critical angle of observation
λ	= wavelength
μ	= $\cos\xi$
ξ	= polar angle
σ_a	= absorption coefficient
σ_s	= scattering coefficient
$\langle\sigma_s p\rangle$	= product of scattering cross section and phase function
ϕ	= angle of incidence
ω	= azimuthal angle
Ω	= solid angle
$\frown$	= upper hemisphere
$\smile$	= lower hemisphere

Subscripts

a	= absorption
e	= extinction
f	= fiber
iso	= isotropic
j	= $1,2,\ldots,M$
m	= medium
s	= scattering
s	= scattered radiation

Superscripts

$(\ )^+$	= positive direction
$(\ )^-$	= negative direction

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Introduction

RADIATION through fiborous materials has been the subject of many recent investigations.[1-10] This is because fibrous media are very effective for thermal insulation. The major modes of heat transfer in fibrous materials are radiation and conduction. It has been pointed out that radiation can account for as much as 50% of the total heat-transfer rate even at room temperatures.[1-3] Hence, much effort has been directed to achieve better understanding of the radiative transfer process in these materials.

In the past, radiation heat transfer through fibrous insulation was calculated using effective thermal conductivities based on empirical models.[4] The effective thermal conductivities, however, all contain a parameter that must be determined experimentally. This limits the practical usefulness of these models for radiative heat-transfer prediction when the boundary conditions and/or the materials are different. Two-flux models were subsequently developed to calculate the radiative heat transfer.[5-7] The basic radiative properties of a single fiber, i.e., the extinction and scattering coefficients, were calculated using well-established formulas for the interaction of electromagnetic radiation with infinitely long circular cylinders.[11,12] The derivation of the two-flux parameters, however, did not properly account for the effect of fiber orientation and the diffuse nature of radiation from diffuse boundaries. The deficiencies of these two-flux models were briefly discussed in a recent paper.[10] It is, of course, important to incorporate both of these effects in order to estimate the radiation heat transfer more accurately.

The problem of radiative transfer through a fibrous medium with fibers randomly oriented in planes for collimated irradiation was presented in a previous paper.[10] The radiative properties of the fibrous medium were shown to be strongly dependent on the fiber orientation. Radiation heat transfer is also expected to be strongly influenced by the fiber orientation. Building insulation materials typically have fibers oriented in random direction in space. Other types of insulation such as those used in spacecraft have fibers that are woven in parallel planes. It is important to be able to estimate the effectiveness of this type of fibrous insulation, as well as the effectiveness for those with randomly oriented fibers.

This paper presents the rigorous formulation of a two-flux model to calculate radiation heat transfer through a fibrous medium with fibers oriented parallel to planar boundaries. Cases of fibers woven in both specific and random azimuthal directions will be considered. The analysis will begin with a general mathematical statement of the problem, followed by the derivations of each of the two-flux parameters.

Analytical Formulation

The present analysis considers only the radiation heat transfer between diffuse planar boundaries. The medium contains fibers oriented in either random or specific directions in planes parallel to the boundaries. The fibrous medium is assumed to be gray and in radiative equilibrium.

The equation of radiative transfer for a gray absorbing, emitting, and scattering medium is

$$\mu\frac{\mathrm{d}I}{\mathrm{d}y} + KI = \frac{1}{4\pi}\int_{\Omega'} \sigma_s p(\Omega' \to \Omega)\,\mathrm{d}\Omega' + \sigma_{am} I_b \tag{1}$$

$$\mu = \cos\xi$$

The energy equation for radiative equilibrium is

$$4\pi I_b \sigma_a = \int_{\Omega} \sigma_{am}(\Omega) I(\Omega)\,\mathrm{d}\Omega \tag{2}$$

where

$$\sigma_a = \frac{1}{4\pi}\int_{\Omega} \sigma_{am}(\Omega)\,\mathrm{d}\Omega \tag{3}$$

Calculation of the two-flux parameters is well established if the extinction, absorption, and scattering coefficients are independent of direction as in the case of spherical particles. In the case of fibers, both the fiber cross section and the backscattering of radiation vary with the relative directions of the incident radiation and fiber orientation. First, the formulation of the radiative properties of the medium is discussed.

Radiative Properties

The radiative properties of the medium are closely related to those of the individual fibers. The fibers in the medium are assumed to be infinitely long circular cylinders. This assumption is justified if the length of the fiber is much larger than both its diameter and the wavelength of the incident radiation. In most insulation, the fibers are usually on the order of 1 mm long and 1–10 μm in diameter, while the characteristic wavelength of thermal radiation is about 10 μm.

The scattering of radiation by a cylinder at an oblique incident angle is shown in Fig. 1. The radiation scattered by a cylinder propagates along the surface of the cone with apex angle $\pi - 2\phi$, where ϕ is the angle of incidence. The direction of the scattered radiation relative to the incident ray is characterized by θ, which is called the angle of observation. The extinction and scattering efficiencies (Q_e and Q_s) for single cylinders are functions of the fiber size, the index of refraction, and the angle of incidence. Typical variation of the efficiencies with the angle of incidence is shown in Fig. 2.

The scattering cross section C_s is related to the unit intensity function $i(\theta,\phi)$ by[11,12]

$$C_s(\phi) = 2rQ_s(\phi) = \frac{\lambda}{\pi^2}\int_0^{2\pi} i(\theta,\phi)\,\mathrm{d}\theta \tag{4}$$

The efficiencies are always highest at normal incidence (i.e., $\phi = 0$). For a fiber in a specific orientation as shown in Fig. 1b, the angle of incidence is related to the incident (ξ,ω) and fiber (ξ_f,ω_f) directions by[10]

$$\cos\left(\frac{\pi}{2} - \phi\right) = \sin\xi\,\sin\xi_f\cos(\omega - \omega_f) + \cos\xi\,\cos\xi_f \tag{5}$$

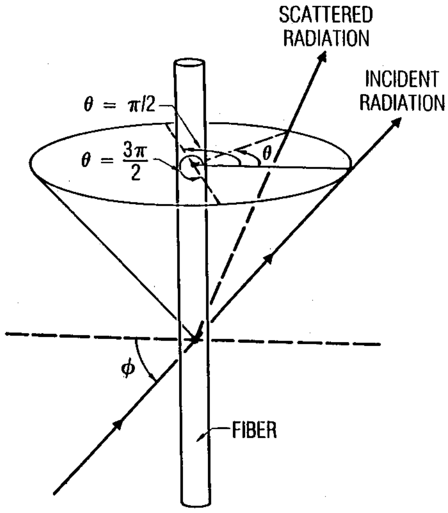

Fig. 1a Scattering of radiation by an infinite circular cylinder.

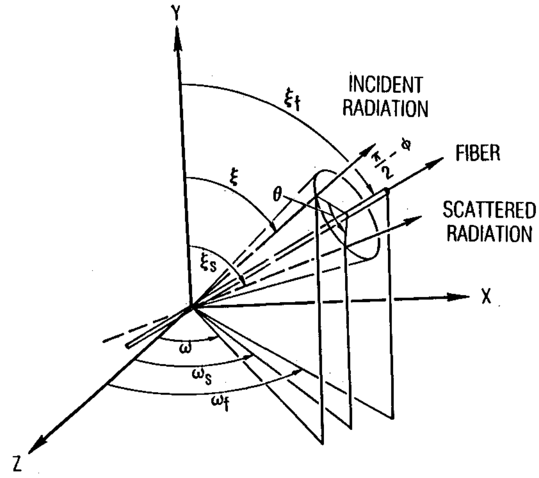

Fig. 1b Fiber orientation in space.

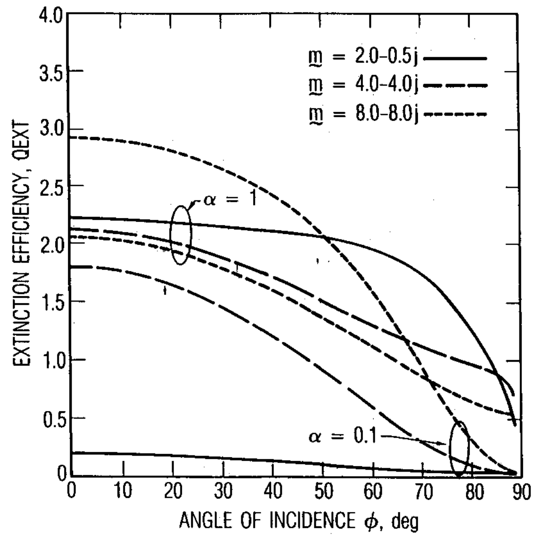

Fig. 2 Variation of the extinction efficiency with the angle of incidence for a single fiber.

Substituting $\xi_f = \pi/2$ for the fiber oriented in a plane, this becomes

$$\sin\phi = \sin\xi\,\cos(\omega - \omega_f) \tag{6}$$

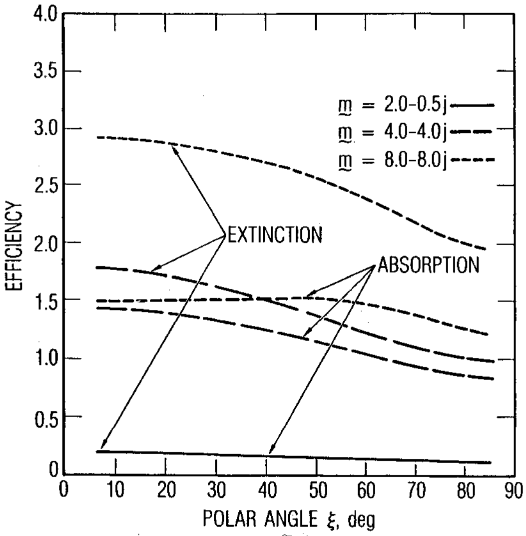

Fig. 3 Variation of the extinction and absorption efficiencies with polar angle for fibers randomly oriented in planes (size parameter $2\pi r/\lambda = 0.1$).

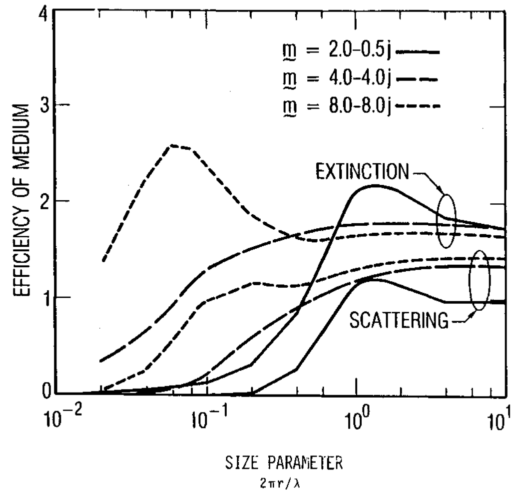

Fig. 4 Effect of fiber size parameter on the effective extinction and scattering efficiencies of the medium.

Fibers Randomly Oriented in Planes

The radiative properties of the medium are obtained by averaging the individual properties using the size and orientation distribution function.[10] For monosize fibers in random azimuthal orientation in planes, the extinction, scattering, and absorption coefficients of the medium (K,σ_s,σ_a) are

$$\{K(\mu),\sigma_s(\mu)\} = 2rN\{Q_{em}(\mu),Q_{sm}(\mu)\}\delta\left(\xi_f - \frac{\pi}{2}\right)$$

$$= \frac{rN}{\pi}\int_0^{2\pi}\{Q_e(\phi),Q_s(\phi)\}\delta\left(\xi_f - \frac{\pi}{2}\right)\mathrm{d}\omega_f \tag{7a}$$

$$\sigma_a(\mu) = K(\mu) - \sigma_s(\mu) \tag{7b}$$

The integration over ω is computed in a straightforward manner by using Eq. (6). Notice that because ϕ varies with $\omega - \omega_f$, the integration over ω_f removes the azimuthal dependence of the radiative coefficients.

The variation of the effective extinction and absorption efficiencies of the medium with polar angles for fibers oriented in planes is shown in Fig. 3. The efficiencies are highest at $\xi = 0$ because this corresponds to normal incidence for all fibers. As the polar angle ξ is increased, the radiative efficiencies decrease because the angle of incidence deviates from the normal direction. Figure 4 shows the effect of fiber size parameter on the effective extinction and scattering efficiencies of the fibrous medium for various refractive indices. The extinction efficiencies all coverge for a large fiber size parameter.

A rigorous derivation of the scattering phase function accounting for fiber orientation was presented by Lee.[10] The general form of the phase function is

$$\langle\sigma_s p\rangle_m = \iint \frac{4\lambda}{\pi^2}\frac{i(\theta,\phi)}{\sin\theta\cos^2\phi}\,d^2F \tag{8}$$

where d^2F is the fiber size and orientation distribution function. This becomes

$$\langle\sigma_s p\rangle_m = \int_0^\infty \int_0^{\pi/2} 2r\left[\frac{4\lambda}{\pi^2}\frac{i(\theta,\phi)}{\sin\theta\cos^2\phi}\right]\cos\phi\,\mathrm{d}\phi N(r)\,\mathrm{d}r \tag{9}$$

for fibers randomly oriented in space. The quantity $N(r)\,\mathrm{d}r$ is the number density of fibers whose radii is between r and $r + \mathrm{d}r$.

Houston and Korpela[8] considered the case of fibers randomly oriented in space and solved the equation of transfer rigorously with the exact phase function. They indicated that the Legendre polynomial expansion of the phase function requires about 20 terms in the far infrared and about 60 terms in the near-infrared. For other fiber orientation, similar series expansions requiring as many terms will have to be determined. The task of determining the appropriate series expansion for the phase function can be a formidable task. For the calculation of diffuse radiation heat transfer using the two-flux model, this difficulty can be circumvented by using an alternate phase function to greatly facilitate computation.

The phase function for a single fiber is defined as

$$p(\theta,\phi') = \frac{i(\theta,\phi)}{\int_0^{2\pi} i(\theta,\phi)\,\mathrm{d}\theta}\,\delta(\phi - \phi') \tag{10}$$

where δ is the Kronecker delta function. Using Eq. (4), the above equation becomes

$$C_s(\phi)p(\theta,\phi) = \frac{\lambda}{\pi^2}\,i(\theta,\phi) \tag{11}$$

Averaging over all fiber orientations, the product of the scattering cross section and phase function for a medium of monosize fibers oriented in planes becomes

$$\langle\sigma_s p\rangle_m(\mu,\omega) = \frac{N\lambda}{2\pi^3}\int_0^{2\pi} i(\theta,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)\mathrm{d}\omega_f \tag{12}$$

Notice that $p(\theta,\phi)$ describes the distribution of the scattered intensity corresponding to the scattering cross section at a particular angle of incidence. Therefore, when the scattering cross section varies with the angle of incidence, C_s and p should always appear in product form. Consequently, both the scattering coefficient and phase function should stay inside the integration over the solid angle as written in Eq. (1). With all the radiative properties properly defined, the two-flux model is next discussed.

Two-Flux Model

The flux equations for the forward and backward directions of the two-flux model can be written as

$$I^+: \mu\frac{\mathrm{d}I^+}{\mathrm{d}y} = -K(\mu)I^+ + \sigma_a(\mu)I_b + FI^+ + BI^- \qquad 0 < \mu \le 1 \tag{13a}$$

$$I^-: \mu\frac{\mathrm{d}I^-}{\mathrm{d}y} = -K(\mu)I^- + \sigma_a(\mu)I_b + BI^+ + FI^- \qquad -1 \le \mu < 0 \tag{13b}$$

where the dependence of the extinction and absorption coefficients on μ is indicated explicitly. This is different from the case of spherical particles whose coefficients are independent of the polar angle. In the above expressions, the forward and backward parameters F and B are

$$F(\mu) = \frac{1}{4\pi}\int_{\frown} \langle \sigma_s p \rangle_m \mathrm{d}\Omega' = \frac{N\lambda}{2\pi^3}\int_{\frown}\int_0^{2\pi} i(\theta,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)\mathrm{d}\omega_f \frac{\mathrm{d}\Omega'}{4\pi} \quad (14a)$$

$$B(\mu) = \frac{1}{4\pi}\int_{\smile} \langle \sigma_a p \rangle_m \mathrm{d}\Omega' = \frac{N\lambda}{2\pi^3}\int_{\smile}\int_0^{2\pi} i(\theta,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)\mathrm{d}\omega_f \frac{\mathrm{d}\Omega'}{4\pi} \quad (14b)$$

Because of the dependence of ϕ on $\omega - \omega_f$, the integration over ω_f removed the aziumthal dependence of these parameters.

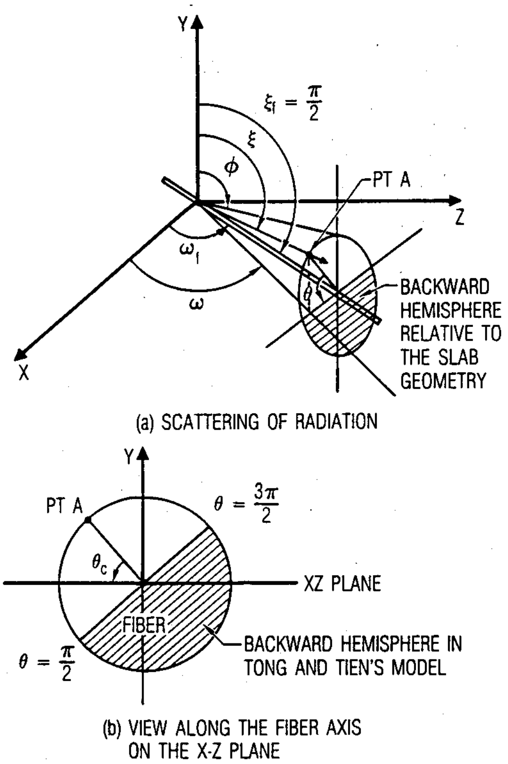

Fig. 5 Radiation scattered by a fiber oriented parallel to the boundaries: a) scattering of radiation; and b) view along the fiber axis on the X-Z plane.

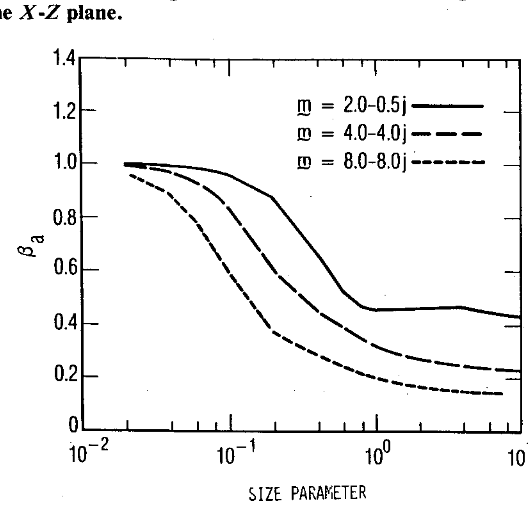

Fig. 6 Absorption ratio for various refractive indices.

Physically, the backward parameter denotes the radiation scattered into the backward hemisphere due to the incident radiation traversing in the forward direction and vice versa for the forward parameter. Hence, the hemispherical integration can be replaced by the integration over the angle of observations encompassing the appropriate half-space, such that

$$F(\mu) = \frac{N\lambda}{2\pi^3}\int_0^{2\pi}\int_{\theta_c+\pi}^{\theta_c+2\pi} i(\theta,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)\mathrm{d}\theta\,\mathrm{d}\omega_f \quad (15a)$$

$$B(\mu) = \frac{N\lambda}{2\pi^3}\int_0^{2\pi}\int_{\theta_c}^{\theta_c+\pi} i(\theta,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)\mathrm{d}\theta\,\mathrm{d}\omega_f \quad (15b)$$

where θ_c is denoted as the critical angle of observation. The geometry of the scattering of radiation by a typical fiber oriented in a plane is depicted in Fig. 5. A ray of radiation at ξ, ω is incident of a fiber oriented at ω_f. The incident radiation intersects the base of the scattered cone of radiation at point A. The critical angle θ_c is the angle of observation measured from point A to the horizontal XZ plane. This angle is given by

$$\theta_c = \cos^{-1}\frac{\sin\xi\,\sin(\omega - \omega_f)}{[1 - \sin^2\xi\,\cos^2(\omega - \omega_f)]^{\frac{1}{2}}} \quad (16)$$

which is derived by setting $\xi_s = \pi/2$ in the transformation in Ref. 10. The parameters F and B satisfy the conservation equation

$$F(\mu) + B(\mu) = \sigma_s(\mu) \quad (17)$$

By integrating Eqs. (13a) and (13b) over the forward and backward hemispheres, respectively, and by using the equation of radiative equilibrium, the net radiative heat flux is

$$q = \frac{e_1 - e_2}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + (\beta_a - 2\bar{B})\bar{K}L} \quad (18)$$

where

$$\bar{K} = \frac{1}{2\pi}\int_{\frown} K(\mu)\,\mathrm{d}\Omega = \int_0^1 K(\mu)\,\mathrm{d}\mu \quad (19a)$$

$$\beta_a = \frac{1}{2\pi\bar{K}}\int_{\frown} \sigma_a(\mu)\,\mathrm{d}\Omega = \frac{1}{\bar{K}}\int_0^1 \sigma_a(\mu)\,\mathrm{d}\mu \quad (19b)$$

$$\bar{B} = \frac{1}{2\pi\bar{K}}\int_{\frown} B(\mu)\,\mathrm{d}\Omega = \frac{1}{\bar{K}}\int_0^1 B(\mu)\,\mathrm{d}\mu \quad (19c)$$

The backscatter factor $\bar{B}$ is equivalent to the product of the scattering albedo and the backscatter factor in the conventional sense. The scattering albedo does not appear explicitly

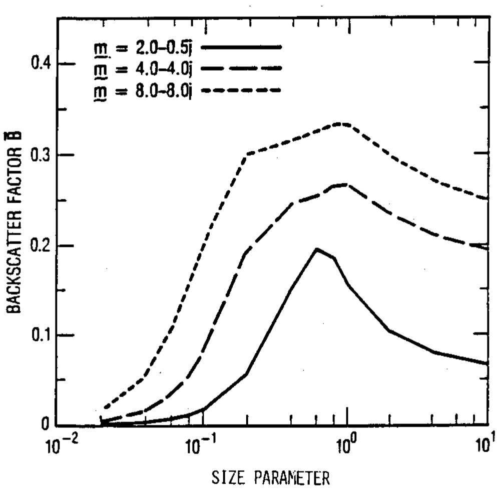

Fig. 7 Backscatter factor for various refractive indices.

because the scattering cross section and the phase function appear as a product. Figures 6 and 7 show the variation of β_a and $\bar{B}$ with the fiber size parameter for several refractive indices. As the fiber size parameter increases, the scattering efficiency also increases which results in the decrease of β_a. The backscatter factor is highest when the size parameters is $\sim \mathcal{O}(1)$.

Fibers Oriented in Specific Direction in Planes

For a fibrous medium containing fibers with a specific orientation in planes, the radiative properties are expressed as the weighted sum of the contribution from the different weave directions. They are[10]

$$\{K(\mu,\omega),\sigma_s(\mu,\omega)\} = 2rN \sum_{j=1}^{M} x_j\{Q_e(\phi),Q_s(\phi)\} \times \delta\left(\xi_f - \frac{\pi}{2}\right)\delta(\omega_f - \omega_{fj}) \tag{20a}$$

$$\langle\sigma_s p\rangle_m(\mu,\omega) = \frac{N\lambda}{\pi^2}\sum_{j=1}^{M} x_j i(\theta,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)\delta(\omega_f - \omega_{fj}) \tag{20b}$$

where M is the number of weave directions and x_j the fraction of fibers oriented in the direction of ω_{fj}. The backward parameter is calculated by integrating the above $\langle\sigma_s p\rangle_m$ over the backward hemisphere similar to that outlined in Eq. (14b). This gives

$$B(\mu,\omega) = \frac{N\lambda}{\pi^2}\sum_{j=1}^{M} x_j \int_{\theta_c}^{\theta_c+\pi} i(\theta,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)\delta(\omega_f - \omega_{fj})\,d\theta \tag{21}$$

The effective coefficients and the backscatter factor are then obtained by integrating Eqs. (20a) and (21) over the hemisphere as

$$\{\bar{K},\bar{\sigma}_s\} = \frac{rN}{\pi}\sum_{j=1}^{M} x_j \int_0^1\int_0^{2\pi}\{Q_e(\phi),Q_s(\phi)\} \times \delta\left(\xi_f - \frac{\pi}{2}\right)\delta(\omega_f - \omega_{fj})\,d\omega\,d\mu \tag{22a}$$

$$\bar{B} = 1 - \bar{\sigma}_s/\bar{K} \tag{22b}$$

$$\bar{\beta} = \frac{N\lambda}{2\pi^3}\sum_{j=1}^{M} x_j \int_0^1\int_0^{2\pi}\int_{\theta_c}^{\theta_c+\pi} i(\theta,\phi)\,d\theta \times \delta\left(\xi_f - \frac{\pi}{2}\right)\delta(\omega_f - \omega_{fj})\,d\omega\,d\mu \tag{22c}$$

The integration over ω removed the dependence of the integrand on fiber direction. The remaining integration(s) can be performed outside of the summation. The above equations then become identical to Eqs. (19) for fibers randomly oriented in planes. This conclusion is also obvious from physical intuition that fibers always appear as randomly oriented with respect to diffuse radiation, regardless of whether specific weave directions exist.

Results and Discussion

With all the parameters rigorously defined, radiation heat transfer through the fibrous medium can be calculated accordingly using the two-flux model. Several refractive indices and fiber size parameters were chosen for demonstration purposes only. Figure 8 shows the variation of the normalized heat flux with the size parameter for a few refractive indices and volume fraction of fibers. As the refractive index is increased (i.e., both real and complex parts), the radiative heat-transfer rate decreased. As the fiber size parameter increases, the heat-transfer rates all converged. This is because the term $(\beta_a + 2\bar{B})$ becomes small compared to utility.

Since two-flux models for fibrous medium have been developed by other investigators,[5,7] it is appropriate to discuss the differences between the current formulation and those of the previous ones. As mentioned before, the previous two-flux models did not account for the dependence of the backscattering and radiative properties on the fiber orientation. Effective radiative properties in these models were calculated by simply averaging over all angles of incidence as $2/\pi \int [\ \ ]\,d\phi$. The resulting values are identical to the efficiencies at the polar angle $\xi = \pi/2$ of the present formulation. When the polar angle dependence of the radiative efficiencies is incorporated into the formulation, the effective extinction efficiency is higher than those obtained by Tong and Tien[5] as shown in Table 1.

The second critical difference between the two-flux models is the proper accounting of the forward and backward directions in the calculation of the backscatter factor. In Tong and Tien's analysis,[5] an integration over all incident angles was first performed on the phase function. Then the backward hemisphere was assumed to be $\pi/2 < \theta < 3\pi/2$ as shown in Fig. 1a. They

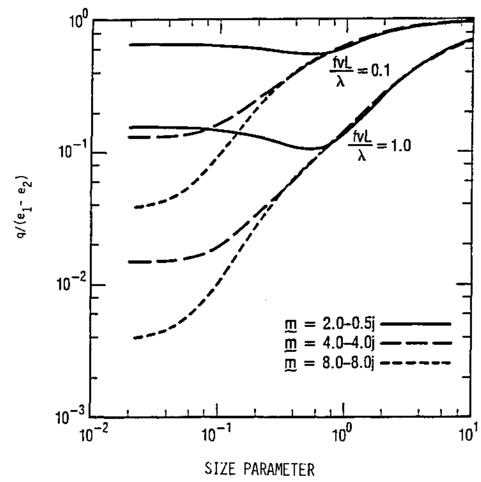

Fig. 8 Variation of the radiative heat flux with the fiber size parameter ($f_v = \pi r^2 \ell N$, ℓ = average length of a fiber, $\varepsilon_1 = \varepsilon_2 = 1.0$).

Table 1 Comparison of effective extinction efficiency and backscattering factor between the current formulation and those of Tong and Tien[5] (m = 4.0 − 4.0j)

Analysis[a]							$2\pi r/\lambda$								
	0.02	0.04	0.06	0.08	0.10	0.20	0.40	0.60	0.80	1.00	2.00	4.00	6.00	8.00	10.00
							$\bar{Q}_e$								
A	0.334	0.649	0.921	1.14	1.29	1.51	1.65	1.71	1.76	1.77	1.78	1.77	1.75	1.74	1.73
B	0.252	0.490	0.699	0.869	0.995	1.24	1.51	1.56	1.57	1.56	1.52	1.48	1.46	1.44	1.43
							$\bar{B}$								
A	0.004	0.015	0.033	0.055	0.080	0.193	0.245	0.253	0.265	0.266	0.235	0.212	0.203	0.198	0.195
B	0.007	0.029	0.060	0.099	0.139	0.295	0.352	0.363	0.374	0.376	0.346	0.333	0.330	0.329	0.328

[a]A = present, B = Tong and Tien.[5]

justified this assumption by noting that the heat-transfer direction is normal to the fiber axes for fibers oriented in planes parallel to the boundaries. This range of θ is the backward direction relative to the incident ray when a single fiber is considered. Because the angle θ denotes the direction of the scattered radiation relative to the incident direction, it is a meaningful parameter only with respect to a combination of incident and fiber directions. It is obvious from Eq. (6) that the angle of incidence can be identical for different combinations of the incident and fiber directions.

For diffuse boundaries, radiation incident on the fibers can emanate from all directions, i.e., $0 < \xi < \pi$ and $0 < \omega < 2\pi$. Hence, there is no single range of θ that can denote the backscattering hemisphere of all incident directions. The computational procedure and the range of θ chosen by Tong and Tien[5] cannot adequately account for the backscattered radiation in the parallel slab geometry as depicted in Fig. 5. Since the angle of incidence on a fiber depends on both the incident direction and the fiber orientation, the forward and backward hemispheres relative to the slab geometry are always different from those relative to a single fiber, except at normal incidence (i.e., $\phi = 0$). For diffuse radiation the angle θ_c must be determined for all incident and fiber directions. Therefore, the previous formulations[5,7] that simply treated the backward direction in the parallel plane geometry as $\pi/2 < \theta < 3\pi/2$ are inadequate. Table 1 shows the comparison of the backscatter factor between the present formulation and that from Tong and Tien.[5] The backscatter factor from Tong and Tien that did not account for the fiber orientation is substantially higher.

The calculation of the backscatter factor in the present formulation involves a triple integration over the angle of observation θ, the azimuthal angle $\omega - \omega_f$, and the cosine of the polar angle μ. In particular, the limits of integration over θ vary with the other two variables. As indicated earlier, the variable limit of integration on θ is necessary in order to identify the backward hemisphere correctly. This resulted from the use of the simple definition of the phase function. Had the phase function been expanded similar to that by Houston and Korpela[8] in terms of the global coordinates, the calculation of the backscatter factor would have been similar to the conventional method, where only a double integration over the polar angles is required. The apparent simplicity in the calculation of the backscatter factor for spherical scatterers is because the phase function has been expanded in terms of the global coordinates. In addition, an integration has also been performed over the azimuthal angle with the assumption of azimuthal symmetry.

It should be pointed out that in the case of fibers, it is not necessary to define effective values for both the scattering coefficient and phase function so as to be able to treat them separately. This is because the fiber cross section varies with the angle of incidence. Since the scattering phase function specifies the distribution of intensity scattered by a particle with a specific cross section, the scattering cross section and phase function should always appear in product form. Defining effective values for both the scattering cross section and phase function would unnecessarily complicate the computations.

Although the current paper considered only gray monosize fibers, the formulation can be easily extended to treat the more general case of nongray optical properties and polydisperse size fibers. For polydisperse fibers, the radiative properties are obtained by simply integrating the respective cross sections and phase function over the size distribution. For nongray optical properties, all the radiative properties are spectrally dependent. The net heat flux is obtained by integrating Eq. (18) over all wavelengths, with the surface emissive powers e_1 and e_2 replaced by the Planck function.

Conclusion

This paper presented a rigorous derivation of the two-flux parameters for the fibrous medium with fibers oriented parallel to diffuse planar boundaries. The effect of the polar and azimuthal orientation of fibers was incorporated into the fomulation. It was shown that radiation heat transfer is independent of the azimuthal orientation of the fibers. Parametric studies were performed to demonstrate the effect of optical properties and the fiber size parameter on the radiative properties and the radiative heat-transfer rates. Based on the current theoretical consideration, it was also possible to point out the deficiencies in the previous two-flux models.

The two-flux model can become inaccurate when scattered radiation is not isotropic. However, as indicated earlier, the more rigorous treatment of solving the equation of transfer would require an appropriate series expansion of the phase function for this particular fiber orientation. This is an extremely tedious task. The complexity is further aggravated when nongray optical properties are considered. The current study used a simpler phase function to facilitate computation. The use of this phase function requires a more detailed consideration of the geometry of scattering. Since the geometry of backscattering has been accounted for rigorously, the present formulation can easily be extended to a multiflux model to improve accuracy. Despite the limitation of a two-flux model, it is nevertheless a simple and useful tool for making engineering estimates.

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